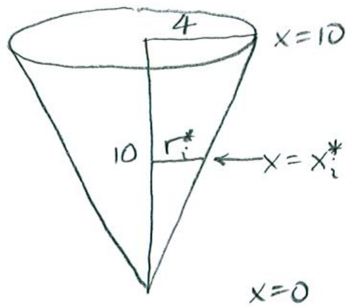
Math 1B
Quiz 6 Version
Fri Nov 9, 2012

NAME YOU ASKED TO BE CALLED IN CLASS: ____________________

SCORE: _____ / 30 POINTS

NO CALCULATORS ALLOWED

A water tank has the shape of an inverted cone (circular base at the top, vertex pointing downward) of height 10 ft radius 4 ft. If the tank is full of water, write, **BUT DO NOT EVALUATE**, an integral for the work done to pump all the water to the top of the tank. You may use ρ as the density of water. **SCORE: _____ / 7 PTS**



SLICE OF WATER ≈ CYLINDER

$d = 10 - x_i^*$

$F = \rho \cdot \pi (r_i^*)^2 \Delta x = \rho \pi \left(\frac{2}{5} x_i^*\right)^2 \Delta x$

$W = \frac{4}{25} \rho \pi (x_i^*)^2 (10 - x_i^*) \Delta x$

$\frac{r_i^*}{4} = \frac{x_i^*}{10}$

$r_i^* = \frac{2}{5} x_i^*$

TOTAL WORK $= \int_0^{10} \frac{4}{25} \rho \pi x^2 (10-x)\, dx$

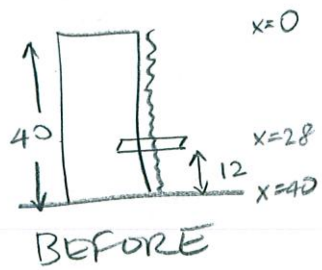
1 1 1 1 1 1 1

$\rho = 62.5$

TALK TO ME IF YOU USED ANY OTHER SCALE

OR $\int_0^{10} \frac{4}{25} \rho \pi x (10-x)^2 dx$

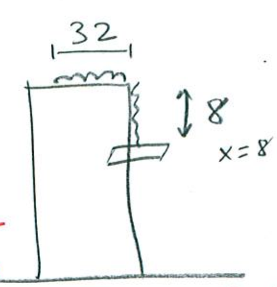
A 40 ft chain weighing 120 lb is hanging from the roof of a 40 ft building. By pulling it towards the roof, the chain is used to lift a 50 lb tabletop from a window 12 ft from the ground to a window 8 ft from the roof. (After the tabletop is lifted to the higher window, the chain is not pulled or lowered any further.) **SCORE: _____ / 7 PTS**
Write, **BUT DO NOT EVALUATE**, an expression involving an integral (or sum of integrals) for the work done.



WORK ON TABLETOP = 50 LB × 20 FT = 1000 FT-LB

32 FT OF CHAIN MOVED FROM $x = 8 \ldots x = 40$ UP TO $x = 0$

WORK ON CHAIN $= \int_8^{40} \frac{120}{40} x\, dx = \int_8^{40} 3x\, dx$



TOTAL WORK $= \left(1000 + \int_8^{40} 3x\, dx\right)$ FT-LB

2 1½ 1½ 1 1

TALK TO ME IF YOU HAD OTHER INTEGRALS

OR $\left(1000 + \int_0^{32} 3x\, dx + (24)(32)\right)$ FT-LB

A continuous random variable X has probability density function $p(x) = \begin{cases} kx^3, & 0 \le x \le 2 \\ 0, & x < 0 \quad or \quad x > 2 \end{cases}$ **SCORE: _____ / 8 PTS**

for some constant k.

[a] Find $P(X > 1)$.

$$\int_0^2 kx^3\,dx = 1 \quad 1$$

$$1 \quad \frac{1}{4}kx^4 \Big|_0^2 = 1$$

$$4k = 1$$

$$k = \frac{1}{4} \quad 1$$

$$P(X>1) = \int_1^2 \frac{1}{4}x^3\,dx \quad 1$$

$$= \frac{1}{16}x^4 \Big|_1^2 \quad \tfrac{1}{2}$$

$$= \frac{1}{16}(16-1) = \frac{15}{16} \quad 1$$

[b] Find the mean value of X.

$$\int_0^2 x \cdot \frac{1}{4}x^3\,dx$$

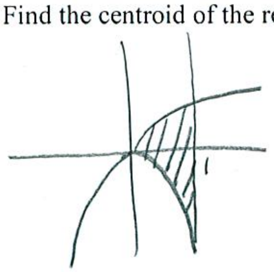
$$= \int_0^2 \frac{1}{4}x^4\,dx \quad 1$$

$$= \frac{1}{20}x^5 \Big|_0^2 \quad \tfrac{1}{2}$$

$$= \frac{1}{20}(32)$$

$$= \frac{8}{5} \quad 1$$

Find the centroid of the region bounded by $y = \sqrt{x}$, $y = -x^2$ and $x = 1$. **SCORE: _____ / 8 PTS**



$$\int_0^1 x(\sqrt{x} - -x^2)\,dx \quad 1$$

$$= \int_0^1 (x^{\frac{3}{2}} + x^3)\,dx$$

$$= \left(\frac{2}{5}x^{\frac{5}{2}} + \frac{1}{4}x^4\right)\Big|_0^1 \quad 1$$

$$= \frac{2}{5} + \frac{1}{4}$$

$$= \frac{13}{20} \quad \tfrac{1}{2}$$

$$1 \quad \frac{1}{2}\int_0^1 \left((\sqrt{x})^2 - (-x^2)^2\right)dx$$

$$= \frac{1}{2}\int_0^1 (x - x^4)\,dx$$

$$1 \quad = \frac{1}{2}\left(\frac{1}{2}x^2 - \frac{1}{5}x^5\right)\Big|_0^1$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{1}{5}\right)$$

$$= \frac{3}{20} \quad \tfrac{1}{2}$$

$$\int_0^1 (\sqrt{x} - (-x^2))\,dx \quad \tfrac{1}{2}$$

$$= \int_0^1 (x^{\frac{1}{2}} + x^2)\,dx$$

$$= \left(\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{3}x^3\right)\Big|_0^1 \quad 1 \quad = \frac{2}{3} + \frac{1}{3} = 1 \quad \tfrac{1}{2}$$

$$\text{CENTROID} = \left(\frac{\frac{13}{20}}{1}, \frac{\frac{3}{20}}{1}\right)$$

$$= \left(\frac{13}{20}, \frac{3}{20}\right) \quad 1$$